Have I got a deal for you!! Ever heard of the Brooklyn Bridge?

CPI

(Consumer Price Index)

From Chapter 17. The Consumer Price Index (Updated 06/2007) a PDF document available from the US Bureau of Labor Statistics at http://www.bls.gov/cpi/#tables

Page 2, last ¶: *"Unfortunately, because the cost of achieving a living standard cannot be observed directly, in operational terms a COLI can only be approximated. Although the CPI cannot be said to equal a cost-of-living index, the concept of the COLI provides the CPI's measurement objective5 and the standard by which we define any bias in the CPI. BLS long has said that it operates within a cost-of-living framework in producing the CPI.*[6] *"*

> [6] *On the use of a cost-of-living index as a conceptual framework for practical decision making in putting together a price index, see Robert Gillingham, "A Conceptual Framework for the Revised Consumer Price Index," 1974 Proceedings of the American Statistical Association, Business and Economic Statistics Section, pp. 46-52.*

Page 3, second ¶: *"Since 1999, the CPI program has used the geometric mean formula to average price change within most item categories. Under certain assumptions that are likely to be true within most categories, an index based on the geometric mean formula will be closer to a COLI than will a Laspeyres index."* **Laspeyres method (arithmetic mean)** compares the price of the old basket of goods q_o for the old and new prices, p_o and p_t

$$P_L = \frac{\sum(p_t \cdot q_0)}{\sum(p_0 \cdot q_0)}$$

Where P_L is the rate of change of the population, p based on an arithmetic ratio. Cancelling q, reduces Laspeyres formula to $P_L = \frac{\sum p_t}{\sum p_0}$

as opposed to the **Törnqvist formula now used by the BLS**. The Törnqvist or Törnqvist-Theil index is the *"geometric average of the n price relatives of the current to base period prices (for n goods) weighted by the arithmetic average of the value shares for the two periods.*[13][14] *"*

$$P_t = \prod_{i=1}^{n} \left(\frac{p_{it}}{p_{i0}}\right)^{\frac{1}{2}\left[\frac{p_{i0}\,q_{i0}}{\sum_{i=1}^{m}(p_{i0}\,q_{i0})} + \frac{p_{it}\,q_{it}}{\sum_{i=1}^{m}(p_{it}\,q_{it})}\right]}$$

*HUH??? P_t is essentially the same rate of inflation as P_L in Laspeyres formula. I used only P_t to indicate **P**rice at **T**ime t, rather than P_L for **P**rice "**L**ater" in the comparative table below. For a Törnqvist index, the growth rates are defined to be the difference in natural logarithms of successive observations of the components (i.e. their log-change) and the weights are equal to the mean of the factor shares of the components in the corresponding pair of periods (usually years). Comparison of methods*

Inflation factor, P_t

(based on original prices of 1, 1, 1, and 1, for 4 items, i)

	Example 1	**Example 2**	**Example 3**
Final Prices	11, 11, 11, 11	11, 11, 11, 111	11, 11, 11, 2
Example Type	Constant change	"expensive" item	"cheap" item
Arithmetic (Laspeyres)	**11.0**	**36.0**	**8.9**
Geometric (Törnqvist)	**11.0**	**38.8**	**8.7**

Compared to the arithmetic method the geometric method overstates the inflation rate for items that increase in value more than average and understates the effects of items that increase less. It all comes out in the wash, *cet. paribus*, except there are usually many more items in any given period that are stable in price (low increase), while a few increase more; then stabilize the next period and others inflate. The overall effect is to understate inflation by 3 to 4 percent relative to the old arithmetic method.

Laspeyres method (pre 1984)

Example 1: changes in price of 10, 10, 10, 10
$q_o = 4$, $p_o = 1, 1, 1, 1$ and $p_t = 11, 11, 11, 11$
P_L = [4(11 x 4)] / [4(1 x 4)]
= 176/16
P_L = 11 times the base period
Example 2: changes in price of 10, 10, 10, 100
$q_o = 4$, $p_o = 1, 1, 1, 1$ and $p_t = 11, 11, 11, 111$
P_L = [3(11 x 4) + 1(111 x 4)] / [4(1 x 4)]
= 576/16
P_L = 36 times the base period
Example 3: changes in price of 10, 10, 10, 1
P_L = [3(11 x 4) + 2(1 x 4)] / [4(1 X 4)]
= 142/16
P_L = 8.9 times the base period

Törnqvist method (post 1984)

Example 1: changes in price of 10, 10, 10, 10
$p_i0 = 1, 1, 1, 1$; $q_i0 = 4$; $p_it = 11, 11, 11, 11$
For i = 1-4: $p_it / p_i0 = 11/1 = 11$, a constant because all $p_it = 11$ and all $p_i0 = 4$
exponential term 1 = 1 x 4 / [4(1 X 4)] = 0.25, a constant because all $p_i0 = 1$
exponential term 2 = 11 x 44 / [4(11 x 44)] = 0.25, a constant because all $p_it = 11$
∴ for each i, $Pt_i = 11^{[0.5 + (0.25 + 0.25)]} = 11^{1.0} = 11$
∴

$$\prod_{i=1}^{n} \frac{p_i t}{p_i o} = (11 + 11 + 11 + 11) / 4$$

P_t = 11.0 times the base period
Example 2: changes in price of 10, 10, 10, 100
$q_o = 4$, $p_o = 1, 1, 1, 1$ and $p_t = 11, 11, 11, 111$
For i = 1-3: $p_it / p_i0 = 11/1 = 11$, a constant because all $p_it = 11$ and all $p_i0 = 4$
exponential term 1 = 1 x 4 / [4(1 X 4)] = 0.25, a constant because all $p_i0 = 1$
exponential term 2 = 11 x 144 / [4(11 x 144)] = 0.25, a constant since $p_it = 11$
∴ for each i, $Pt_i = 11^{[0.5 + (0.25 + 0.25)]} = 11^{1.0} = 11.0$
For i = 4: $p_it / p_i0 = 111/1 = 111$
exponential term 1 = 1 x 4 / [4(1 X 4)] = 0.25, the same as above
exponential term 2 = 111 x 144 / [3(11 x 144) + 1(111 x 144)]
= 15984 / (4752 + 15984)
= 0.77
∴ for $Pt_4 = 111^{[0.5 + (0.25 + 0.77)]} = 111^{1.02} = 122.0$

$$\prod_{i=1}^{n} \frac{p_i t}{p_i o} = (11 + 11 + 11 + 122) / 4$$

P_t = 38.8 times the base period
Example 3: changes in price of 10, 10, 10, 1
$q_o = 4$, $p_o = 1, 1, 1, 1$ and $p_t = 11, 11, 11, 2$
For i = 1-3: $p_it / p_i0 = 11/1 = 11$, a constant because all $p_it = 11$ and all $p_i0 = 4$
exponent term 1 = 0.25, as Examples 1 and 2
exponent term 2 = 0.25, as Examples 1 and 2
∴ for each i, $Pt_i = 11^{[0.5 + (0.25 + 0.25)]} = 11^{0.25} = 11.0$, as Examples 1 and 2
For i = 4: $p_it / p_i0 = 2/1 = 2.00$
exponential term 1 = 1 x 4 / [4(1 X 4)] = 0.25, the same as above
exponential term 2 = 2 x 35 / [3(11 x 35) + 1(2 x 35)]
= 70 / (1155 + 70)
= 0.057
∴ for $Pt_4 = 2^{[0.5 \times (0.25 + 0.06)]} = 1.0$
$= 2^{0.81}$
= 1.75

$$\prod_{i=1}^{n} \frac{p_i t}{p_i o} = 11 + 11 + 11 + 1.75 /4$$

P_t = 8.7 times the base period

Hedonistic Adjustment

From http://www.bls.gov/cpi/cpihqaqanda.htm

Frequently Asked Questions about Hedonic Quality Adjustment in the CPI

To derive the estimated price of Item B', we use the following equation:

$$P_{B,t+s-1} = P_{A,t+s-1} * e^{(\Sigma \beta B - \Sigma \beta A)}$$

Where PB,t+s-1 is the quality adjusted price, PA,t+s-1 is the price of Item A in the previous period, and is the constant e, the inverse of the natural logarithm, exponentiated by the difference of the summations of the ßs for the set of characteristics that differ between items A and B. The exponentiation step is done to transform the coefficients from the semi log form to a linear form before adjusting the price. For our television example, equation looks like this:

$$P_{B,t+s-1} = \$250.00 * e^{(\Sigma(42*screensize-1764*screensize^2 + plasma+HDTV)-\Sigma(27*screensize-729*screensize^2 + EDTV))}$$

$$= \$250.00 * e^{(\Sigma(3.50616-0.87748+0.72483+0.34280)-\Sigma(2.25396-0.36263+0.12228))}$$

$$= \$1345.02$$

When this quality adjustment is applied, the ratio of price change looks like this:

Characteristics	**Item A**	**Item B**
Price	$1,345.02	$1,250.00
Features	42 Inch	42 Inch
	Plasma	Plasma
	HDTV	HDTV
	S-Video Input	S-Video Input
	Universal Remote	Universal Remote

The resulting price change is -7.1 percent after the quality adjustment is applied.